High accuracy speed measurement using GPS (Global Positioning System)

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Abstract.
This article demonstrates that speed measurement with accuracy approaching 1 knot is possible by using GPS Doppler data The method is illustrated with measurements made by GT-11 GPS unit made by Locosys

Introduction
Typical approach to using GPS for speed measurement today is to consider a series of trackpoints that record position estimates latitude and longitude determined by the GPS at regular time intervals

Each GPS trackpoint is determined with some error that is variable and difficult to determine Hence, speed values computed from a series of trackpoints have unknown accuracy and cannot be considered reliable It is virtually *impossible* to prove the accuracy of speed computed from a recorded series of trackpoints

The most inaccurate is the method that tries to estimate an average speed over some accumulated distance between trackpoints Due to trackpoint inaccuracies, the line connecting all track points is a zig-zag, even if the real path of a speed competitor is a smooth or straight line Since the length of this zig-zag is always longer than a smooth ~~straight~~ line, the average speed determined with the accumulated distance method *always overestimate*s the real speed The less accurate are trackpoints the less accurate is a GPS unit – the larger the estimated average speed and the more impressive is the achievement

Doppler
An alternative to measuring speed from series of trackpoints is using the Doppler effect

Modern GPS devices implement digital PLL phase-lock loop receivers to continuously track carrier frequencies of a number of satellites For example, GT-11 tracks carrier frequencies of up to 1 satellites simultaneously The frequency tracking *has to* be continuous simply because each receiver has to be *always ready* to receive data from its satellite

The very fact that data is read from any given satellite is proof that its carrier frequency is tracked with high accuracy

The difference between the known satellite carrier frequency and the frequency determined at the receiver is known as a Doppler shift This Doppler shift is *directly proportional* to velocity of the receiver along the direction to the satellite, regardless of the distance to this satellite

With multiple satellites tracked it is possible to determine the D velocity vector of the receiver In general, the more satellites are tracked – the better the speed estimate

Accuracy of the Doppler tracking
The Doppler speed measurement accuracy is not constant It depends on the number of tracked satellites as well as on their geometrical distribution above the horizon

For this reason it is important to measure this accuracy directly - together with the actual speed determined from the Doppler shift if possible

The most convenient way of verifying the Doppler speed measurement accuracy is recording the Doppler speed data of a stationary GPS receiver at regular time intervals This method accounts for all known[6] GPS-Doppler speed measurement errors Results of 1 hour recording using GT-11 are presented in Fig 1

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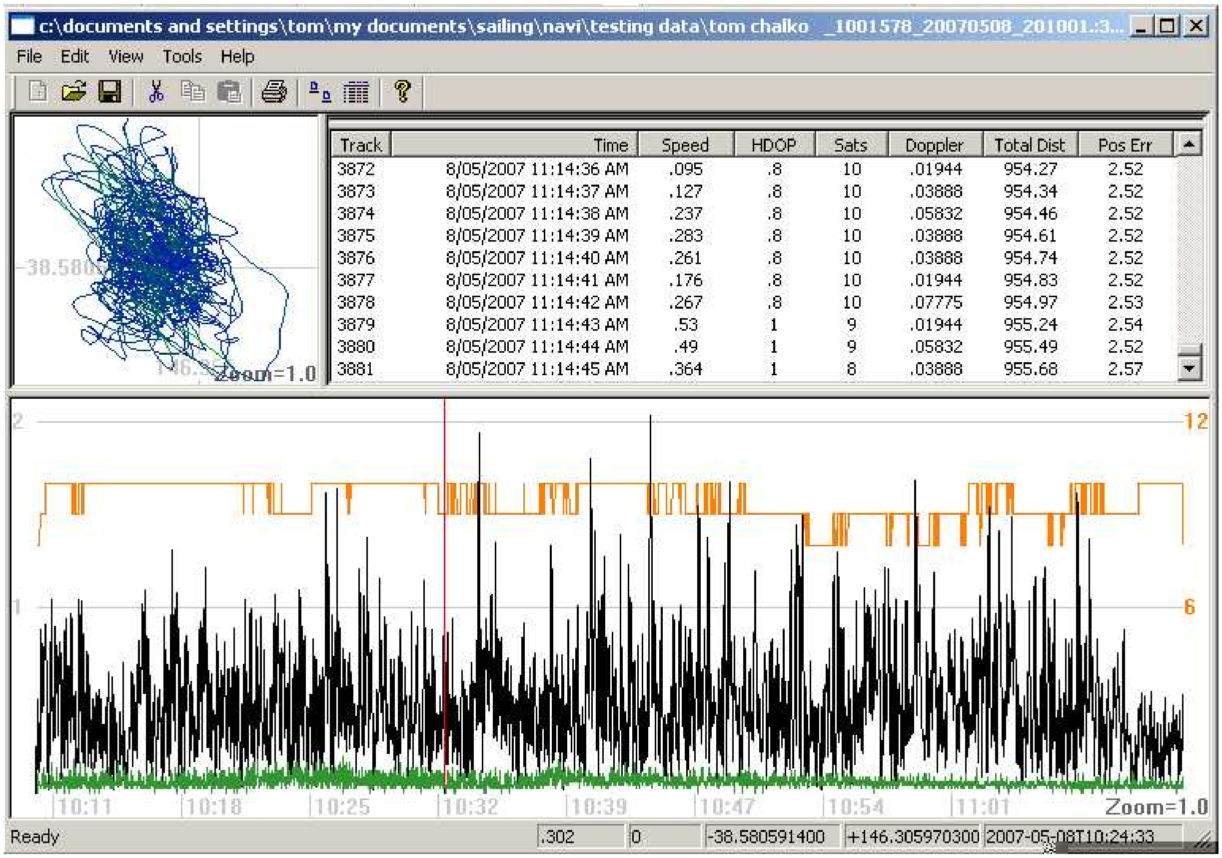


Fig.1. Comparison of Doppler speed and trackpoint-derived speed for stationary GT-11 using RealSpeed software. Binary GT-11 data was logged for about 1 hour every second. Top-left: latitude-longitude trackpoint trajectory. Top-right: a fragment of the data file (distance in meters, speed in knots). Bottom: speed in knots: black – speed determined from trackpoints, green: Doppler speed, orange: number of tracked satellites. Horizontal axis shows time in hours:minutes. The measured average Doppler speed is 0.0554 knot. The position-derived average speed is 0.479 knots. The GPS unit was stationary, but the accumulated distance computed from positions (trackpoints) is almost 1 km. Note the difference between HDOP (Horizontal Dilution of Precision) and the gps-chip computed Position Error for each trackpoint.

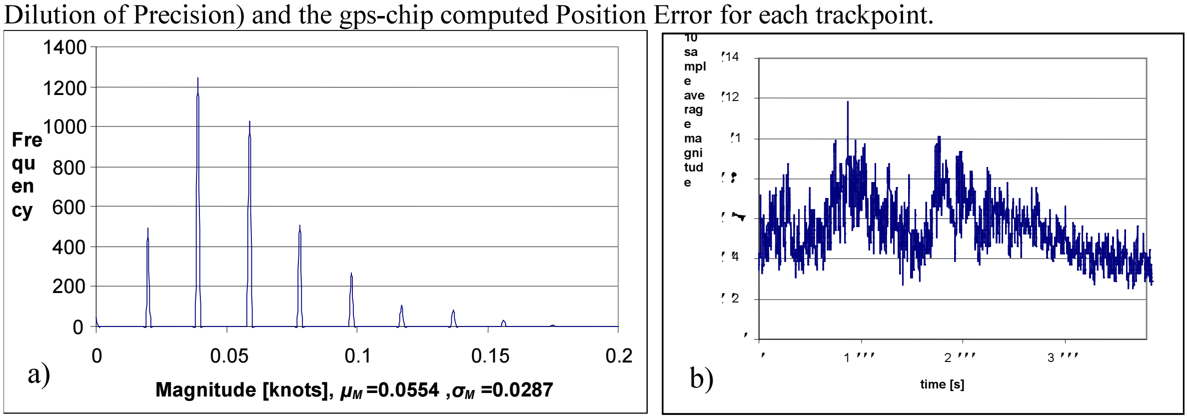


Fig.2. a) Measured distribution of the magnitude of Doppler measurement error in 1-second samples during 1-hour observation. Discrete resolution is clearly visible with 1-bit corresponding to about 0.0194 knot (0.01 m/s). b) Doppler measurement error as a function of time. 10-sample average error magnitude is shown. 1-bit resolution is constant (a), but the Doppler measurement accuracy varies in time.

Doppler accuracy at non-zero speed

What will happen to the Doppler speed resolution and accuracy for non-stationary GPS

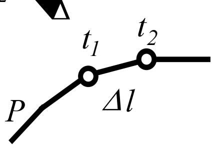
We need to remember that Earth spins A point that seems stationary with respect to the surface of the Earth actually moves West-to-East with speed in the order of 7 knots in the geocentric frame of reference knots at Sandy Point, Vic, Australia Yes, every parked car can receive a speeding fine

Hence, from the point of view of the PLL Doppler frequency tracking system implemented in GPS devices it actually does not matter whether we try to measure speed of 7 knots, knots or 7 knots

Improving the accuracy by averaging

In their wisdom, developers of GT-11 enabled continuous logging of low-pass filtered Doppler speed data at 1 second intervals This enables us to evaluate the average speed with greater accuracy than the accuracy of a single Doppler sample

Let's begin with recalling the universally accepted definitions of speed and the average speed Consider object moving along path *P* and traveling the distance *l* during the time interval $t = t_2 - t_1$



The speed *v* of the object is defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta l}{\Delta t} = \frac{dl}{dt} \quad 1$$

The average speed $\hat{v}$ over time *T* is defined as a time average of *v* as follows

$$\hat{v} = \frac{\int_{t=0}^{T} v dt}{T} = \frac{\int_{0}^{L} dl}{T} = \frac{L}{T}$$

Definition of the average speed determines unique relationship between the travelled distance *L* measured along the path *P* and the average speed $\hat{v}$

$$L = \hat{v} T$$

It is important to point out that the average speed over the time *T* and the average speed over the distance *L* are identical: *there exists only one average speed that describes motion* For this reason, the average speed is the best possible measure of speed in sports that use speed for their ranking

In real measurements, each discrete GPS-Doppler speed sample v_{kD} contains measurement error v_{ke} Hence true speed v_k is $v_k = v_{kD} - v_{ke}$ For a discrete series of speed samples v_{kD} acquired at *N* uniform time intervals over the time *T* we can approximate the integral in the equation by a sum and the expression for the unknown average speed becomes

$$\hat{v} = \frac{\sum_{k=1}^{N} v_{kD}}{N} - \frac{\sum_{k=1}^{N} v_{ke}}{N} = \hat{v}_D - \hat{v}_e \quad 4$$

The first term $\hat{v}_D$ is the exact average of all measured Doppler samples the second term $\hat{v}_e$ represents the measurement error Other methods of integration of the discrete series are discussed later in this article

Measurement errors v_{ke} of each sample can be considered a series of independent random variables that share the same probability distribution *D* Fig demonstrates that both the expected value μ and the standard deviation σ of *D* exist and are finite A typical measured distribution of the magnitude of the speed error v_{ke} is shown in Fig In reality, the probabilities of errors v_{ke} increasing and decreasing the measured values of speed are equal Hence each random variable v_{ke} can be considered to have $\mu =$ and $\sigma < \sigma_M + \mu_M$, where μ_M is he expected value and σ_M is the standard deviation of the measured magnitude of the speed error v_{ke}

The standard error $\sigma_{\hat{v}}$ of the measured average speed $\hat{v}$ can be determined directly from the Central Limit Theorem[5] to be

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \frac{\sigma}{\sqrt{N}} < \frac{\mu_M + \sigma_M}{\sqrt{N}}$$

This error is inversely proportional to $\sqrt{N}$ and hence it decreases with the number of sampling intervals As the number of sampling intervals *N* increases, the distribution of $\sigma_{\hat{v}}$ approaches the normal distribution, enabling us to determine the claimed average speed $\hat{v}$ with any required confidence level

In the absence of measured μ_M and σ_M the standard error σ specified by the GPS manufacturer for Doppler speed should be used

Non-uniform sampling intervals

When multiple GT-11 units are used to sample GPS-Doppler speed at 1Hz each, it is possible to synchronize their sampling sequences to achieve GPS-Doppler speed samples at sub-second intervals, because all GT-11 units report their samples in UTC time with 1ms accuracy Initially set time skewing has been found maintained for many hours providing that all synchronized GT-11 units kept tracking satellites and hence provides us with a method to increase the GPS-Doppler sampling frequency

Ideally, when K GT-11 units are used, each second should be divided into K equal sampling intervals However, in practice, K sub-second intervals may not be equal For this reason we need to determine the average speed and its standard error for such a situation

Without loss of generality we may consider the case of $K=2$ GT-11 units and the corresponding sub-second time intervals Δt_1 and Δt_2, because as we shall soon see the $K=2$ case is extendable to any K The integral in equation ( ) can be approximated by a discrete sum of samples as follows

$$\hat{v} = \frac{\sum_{k=1}^{N_1} v_{kD}\Delta t_1 + \sum_{k=1}^{N_{21}} v_{kD}\Delta t_2}{N_1\Delta t_1 + N_2\Delta t_2} + \\ - \frac{\sum_{k=1}^{N_1} v_{ke}\Delta t_1 + \sum_{k=1}^{N_2} v_{ke}\Delta t_2}{N_1\Delta t_1 + N_2\Delta t_2} = \hat{v}_D - \hat{v}_e \qquad (6)$$

where N_1 and N_2 are numbers of sampling intervals Δt_1 and Δt_2 respectively in T and $N = N_1 + N_2$ The first term represents $\hat{v}_D$ the exact average of all measured Doppler samples the second term represents $\hat{v}_e$ the measurement error Applying the Central Limit Theorem[5] to each term in the numerator of $\hat{v}_e$ and implementing the theorem governing addition of variances of independent variables[8] we obtain the following expression for the standard error $\sigma_{\hat{v}}$ of the average speed $\hat{v}$:

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \sigma\frac{\sqrt{N_1\Delta t_1^{\,2} + N_2\Delta t_2^{\,2}}}{N_1\Delta t_1 + N_2\Delta t_2} \qquad (7)$$

This formula is easily extendable to accomodate K diffferent sampling intervals as follows

$$\sigma_{\hat{v}} = \sigma_{\hat{v}_e} = \sigma\frac{\sqrt{\sum_{k=1}^{K} N_k\Delta t_k^{\,2}}}{\sum_{k=1}^{K} N_k\Delta t_k}.$$

For $K=1$ this formula gives equation ( ) with N being the number of sampling intervals Δt during the time T

Rectangular rule of integration

So far we approximated the intergral in the equation ( ) by a simple sum of discrete terms This method of integration is referred to as a rectangular rule of integration [8] In this rule it is assumed that the integrated function does not change between samples

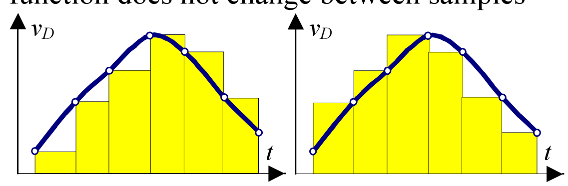


Fig 3. Comparison of Left Riemann and Right Riemann approximation in the rectangular rule of integration.

For a large number of discrete samples Right Riemann, Left Riemann approximations as well as midpoint approximation that is not a convenient option for samples of an unknown function methods converge to the same result, but it is clear that for a limited number of experimentally acquired samples the rectangular rule of integration can be a source of ambiguity

Trapezoidal Rule of integration

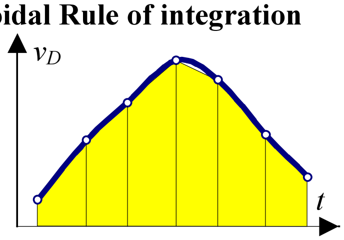


Fig 4. Trapezoidal rule of integration offers excellent accuracy for a limited number of discrete samples and eliminates the ambiguity associated with the rectangular rule of integration

Integration accuracy of an unknown function represented by a limited number of discrete samples can be improved and ambiguity associated with the rectangular rule of integration can be eliminated by implementing the so-called trapezoidal rule of integration [8]

It is important to point out that in the rectangular method of integration the number of GPS-Doppler samples N is equal to the number of the sampling intervals during the time T see Fig and all samples are given the same weight

In contrast, *the trapezoidal rule requires one extra sample and requires the first and the last sample in the interval T to be given weight of 0.5*

For a uniform sampling period *K=1* the trapezoidal rule can be expressed as follows

$$\hat{v}_D = \frac{\sum_{k=1}^{N} v_{kD}}{N} - \frac{v_{0D} + v_{ND}}{2N}$$

where v_{0D} and v_{ND} are the first and the last samples in the integration interval T The expression for $\hat{v}_D$ in the general case of K durations of sampling intervals is

$$\hat{v}_D = \frac{\sum_{m=1}^{K} \sum_{k=1}^{N_K} v_{kD} \Delta t_m}{\sum_{k=1}^{K} N_k \Delta t_k} - \frac{v_{0D}\Delta t_1 + v_{ND}\Delta t_K}{2\sum_{k=1}^{K} N_k \Delta t_k} \qquad 1$$

where $N = \sum_{k=1}^{K} N_k$, and Δt_1 and Δt_K are the corresponding the first and the last sampling intervals

Using the same tools as in the previous section of this article we can estimate the standard error $\sigma_{\hat{v}}$ of the unknown average speed $\hat{v}$ obtained using the trapezoidal rule of integration to be

$$\sigma_{\hat{v}} = \sigma \frac{\sqrt{\sum_{k=1}^{K} N_k \Delta t_k^{\,2} - \frac{\Delta t_1^{\,2}}{2} - \frac{\Delta t_K^{\,2}}{2}}}{\sum_{k=1}^{K} N_k \Delta t_k} \qquad 11$$

For *K=1* this formula gives the equation providing that N_k is the number of sampling intervals Δt_k during the time T

Simpson Rule of integration

The so-called Simpson rule[8] of integration of discrete series of samples uses parabolas to interpolate between discrete samples and is regarded as more accurate than the trapezoidal rule Although it is possible to use the Simpson rule to compute $\hat{v}_D$ and estimate the standard error $\sigma_{\hat{v}}$ of the corresponding average speed, the gain in the accuracy is likely to be too small to be worth an effort

Average of *M* speed attempts

In some sports ranking is based on the average speed of several attempts runs If the number of attempts is M , the standard error of their average speed is

$$\sigma_{\hat{v}_e} = \frac{1}{M}\sqrt{\sum_{k=1}^{M} \sigma_k^2} \ , \qquad 1$$

where σ_k is the standard error of the average speed of the *k*-th attempt

Claimed speed

In sports that base their rankings on the achieved average speed it is convenient to introduce the concept of claimed average speed , being the average speed that can be claimed to have been achieved with the required level of confidence in the presence of measuring errors Let's define the claimed speed as

$$\hat{v}_{CLAIMED} = \hat{v}_D - c\,\sigma_{\hat{v}} \qquad 1$$

Fig illustrates the fact that by adopting *c=2* we can claim with 97.725% confidence that the claimed average speed has been achieved

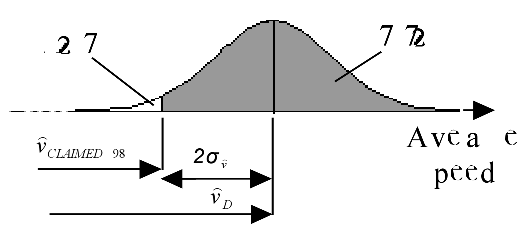


Fig. 5. Relationship between the measured average speed $\hat{v}_D$ and claimed speed for c =2

The graph in Fig illustrates relationship between the coefficient *c* and the corresponding confidence level of the claimed speed

It is clear that if we ignore measurement errors ($c=0$) we can claim with confidence that $\hat{v}_D$ has been achieved

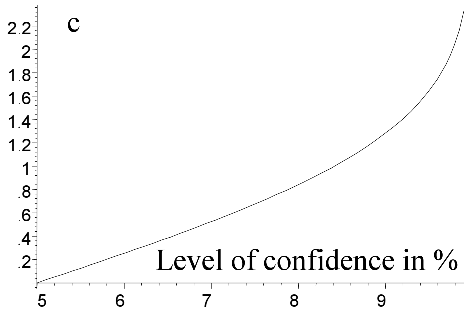


Fig 6. Relationship between the coefficient *c* and the confidence level of the claimed speed in the range 50% to 99% for normal distribution of $\sigma_{\hat{v}}$

Increasing *N*

Regardless of the method of integration used, the accuracy of the average speed increases with the total notal t t ale , I R qAFRF, I q]W F AFW,tl z,[sIFqA R,peI FqARR, chiI qAxx , The variability range of the standard speed error σ is not large, but observable In two days of experimenting near Sandy Point, Victoria, Australia I observed the average Doppler standard speed error σ values ranging from to 1 knots Fig 1 bottom and Fig b illustrate this variability

One way to determine the actual accuracy of Doppler speed measurement during any given event is to use identical GPS units like GT-11 for the entire duration of the event, one unit placed in a stationary location and the other placed on a moving craft or person The theoretical basis for this procedure is the fact that reasons for Doppler signal errors[6] are common for multiple units that are sufficiently close to one another and hence the corresponding speed error distrubutions are correlated

Care should be taken that a similar number of satellites are visible from both units In case of speed windsurfing for example this is equivalent to the requirement of installing the GPS unit on top of a helmet so that is never under water

An alternative method would be to use one GPS unit, but make it motionless for a few minutes before and after each speed attempt

Doppler used in track data
It is important to note, that m s 1 knot accuracy in speed over 1 second interval is equivalent to cm track position accuracy

GPS chipset developers know this very well and actually use Doppler speed measurement to improve the accuracy of trackpoints

Unfortunately, algorithms that mix Doppler and satellite distance data are proprietary and their unknown properties cannot be used to prove the speed or position

Aliasing
The accuracy analysis presented above in this article is valid only when GPS-Doppler speed samples are free from aliasing errors

Aliasing may occur if the bandwidth of the sampled Doppler process is too high in comparison to the sampling frequency The Nyquist criterion requires the sampling frequency to be at least twice the maximum frequency present in the sampled process

When a sattelite phase is steady enough to admit this satellite data for position and speed calculations, the bandwidth of a typical Phase Lock Loop filter that tracks this satellite signal is Hz

This means that in order to eliminate aliasing errors the following solutions are available

1 Use 4 Hz GPS-Doppler sampling rate This can be accomplished ether by using one GPS unit that is capable of reporting Doppler samples at 4 Hz or four 1Hz GPS units synchronized to provide Doppler samples approximately every ms
When sampling frequency is limited to F for example Hz for twin GT-11 units, mechanical motion of the GPS unit used to measure speed should not contain frequencies above $F/2$ 1Hz in our example. Mechanical vibrations of the GPS unit with frequencies above $F/2$ above 1Hz in our example should be filtered either using a seismic suspension or biofeedback locating the GPS on a head of the competitor

Implementation of the second solution is described in the Reference 7

Spoofing
Opponents of direct measurements of average speed claim that it is possible to boost the average speed result by inducing transverse oscillations and thereby increasing the length L of the travelled trajectory Let's explore the upper limit of this boost in the sport of speed windsurfing.

Consider transverse harmonic oscillations with peak-to-peak stroke S [m], and frequency f [Hz] superimposed on motion with velocity V [knots] When speed is sampled with frequency F, the maximum frequency of oscillations that can be recorded is $f=F/2$

If oscillations are optimally phase-synchronized with the sampling process, samples will contain the maximum possible average transverse velocity *SF* during the stroke The corresponding speed boost ε_B is

$$\varepsilon_B = \sqrt{V^2 + (1.944SF)^2} - V \quad \text{[knots]},$$

where the factor 1 44 converts m~~s to kno~~ts For sampling rate *F*=1 Hz and speed *V*4 knots this means that a machinery of 1m in size that generates motion precisely phase synchronized with the GPS sampling process is required to boost the average speed by just knots Such machinery is not only illegal in sailing, but will introduce extra drag and hence reduce the sailing speed

For many reasons such as satellite visibility, signal strength, reliability and accuracy of GPS measurements it is recommended that speed-recording GPS units to be worn on ~~inside~~ a helmet[7] worn by a sailor The stroke *S* of human head is limited to about 1 m The corresponding upper limit of speed boost that can be achieved at *V*4 knots is 4 knots

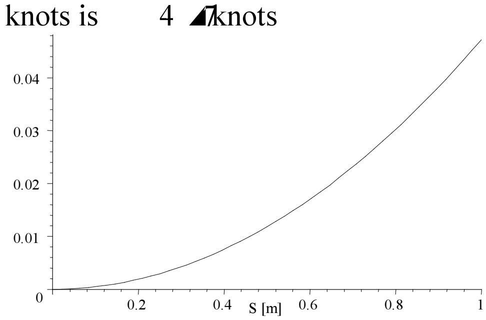


Fig 7. Maximum possible speed boost [knots] as a function of stroke for 1Hz speed samples at *V*=40 knots.

At World Record speed *V*= this upper limit for the speed boost is knots

Let's now explore increasing the frequency of oscillations *f* to increase the speed boost Human head cannot move with frequency faster than Hz and stroke larger than about cm m Due to the upper frequency limit, the maximum possible speed boost would be achieved when speed is sampled at 4 Hz The corresponding maximum physiologically achievable speed boost at *V*= knots would be 1 knots Speed sampling at rates higher than 4 Hz will actually reduce the average speed boost, because not all samples used in the average speed calculation will contain the full amount of the boosted speed

In view of the above analysis we have to conclude that, providing that aliasing is eliminated, effects of transverse oscillations can be considered insignificant for speeds above knots when speed recording GPS units are installed on ~~inside~~ helmets of windsurfing sailors

Future of GPS technology

Today GT-11 unit provides 1 m~~s speed~~ measurement resolution and 1 m~~s accura~~cy for each one-second sample

The short term future of hand-held GPS devices can be predicted by studying specifications of newly released GPS microprocessor chipsets that haven't yet found their way to the mass market

These specifications indicate that as soon as we can expect GPS devices offering 1 m~~s Dopp~~ler speed accuracy for each sample It is rather interesting that the positional trackpoint acurracy will remain around meters

This seems yet another significant argument to commit to Doppler method for speed measurement

Conclusions

- Doppler shift is directly proportional to speed Hence, measuring Doppler shift is the most direct and hence the most accurate way of measuring speed
- Doppler frequency is relatively insensitive to distances from satellites, phase delays and many other factors[6] that are major sources of errors for trackpoints
- Doppler method of speed measurement provides proof of speed, because it is trackable to Units of Measurement
- Tolerance and accuracy of Doppler measurement can easily be measured and monitored experimentally

- Accuracy of Doppler speed measurements can be significantly improved by adopting **the average speed** as a measure of speed
- Accuracy of the average speed measurement over given amount of time interval increases with the number of Doppler speed samples in this interval
- **Very high accuracy** in the average speed measurement can be achieved today providing that sufficiently large number of suitable GPS instruments GT-11 is used to measure the speed
- Doppler speed measurements are repeatable and reproducible with experimentally verifiable accuracy and resolution
- The most practical method of computing the average speed from GPS-Doppler samples is to use the trapezoidal rule of integration

GPS Doppler tracking data from multiple satellites provides a very accurate and very easy way of measuring average speeds The longer the measuring period is, the more frequent are Doppler speed samples and the more GPS units are used simultaneously to measure the same speed - the better the accuracy

For a speed event of seconds or more, the average speed measurements with accuracy as high as 1 knot is possible with technology that exists **today** the GT-11 unit from Locosys

Are we ready to know the Real Speed we achieve

Really

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